UNIT 3 REVIEW #2: PHOTON THEORY (SOLUTIONS)

1. a) 3.20×10^{-15} J $\times \dfrac{1\,eV}{1.60\times10^{-19}\,J}$ = 20,000 eV = 20.0 keV

   b) 67.0 GeV = 67.0×10^{9} eV $\times \dfrac{1.60\times10^{-19}\,J}{1\,eV}$ = 1.07×10^{-8} J

2. Find the wavelength:

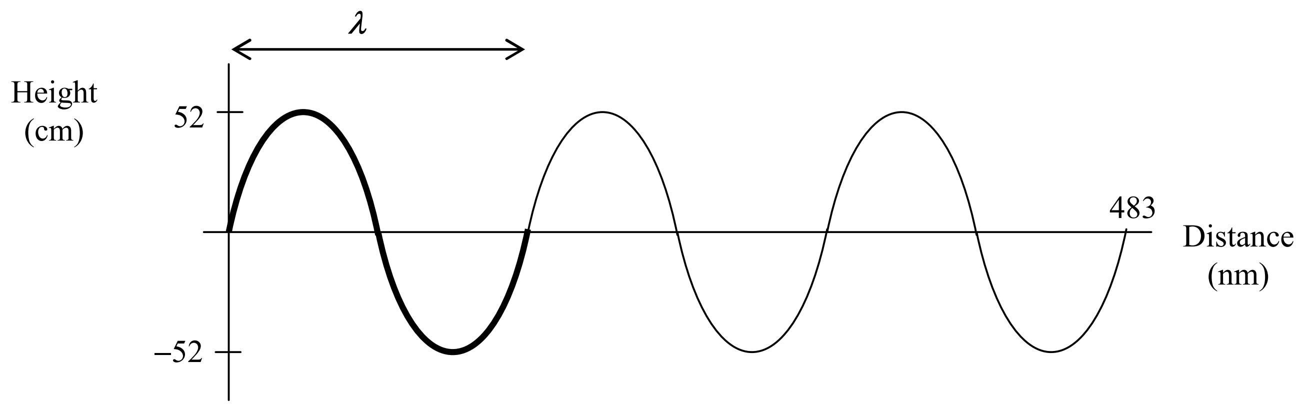
IR	
R	700 nm
O	
Y	
G	
B	
I	
V	400 nm
UV	

$$E_{photon} = \frac{hc}{\lambda} \qquad E\,\lambda = h\,c \qquad \lambda = \frac{hc}{E}$$

$$\lambda = \frac{(6.63\times10^{-34}\,Js)(3.00\times10^{8}\,m/s)}{2.19\times10^{-19}\,J} = 9.08 \times 10^{-7}\text{ m} = 908\text{ nm}$$

Since the wavelength is longer than 700 nm, this is infrared EMR.

3.



Based on the diagram above, we see that $\lambda = \frac{1}{3}\left(483\times10^{-9}\,m\right)$ = 1.61×10^{-7} m

$$E_{photon} = \frac{hc}{\lambda} = \frac{(4.14\times10^{-15}\,eVs)(3.00\times10^{8}\,m/s)}{1.61\times10^{-7}\,m} = 7.714\text{ eV}$$

Finally, $E_T = n\,E_{photon}$ $\qquad n = \dfrac{E_T}{E_{photon}} = \dfrac{800\,eV}{7.714\,eV}$ = 104 photons

4. The minimum energy to remove an electron is called the work function.
   i.e. $W = 4.13 \times 10^{-19}$ J

   Also, based on $v = f\lambda$ $\lambda = \frac{c}{f}$ $\lambda \propto \frac{1}{f}$ Inverse relationship

   Thus, when wavelength is at a maximum, frequency is at a minimum (i.e. threshold frequency f_0).

   So, $W = h f_0$ $f_0 = \frac{W}{h} = \frac{4.13\times10^{-19} J}{6.63\times10^{-34} Js}$ = 6.229×10^{14} Hz

   $$\lambda_{\max} = \frac{c}{f_0} = \frac{3.00\times10^{8} m/s}{6.229\times10^{14} Hz} = 4.82 \times 10^{-7} \text{ m} = 482 \text{ nm}$$

5. $W = h f_0 = (6.63\times10^{-34} Js)(5.5\times10^{14} Hz) = 3.6465 \times 10^{-19}$ J

   $$E_{photon} = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34} Js)(3.00\times10^{8} m/s)}{380\times10^{-9} m} = 5.2342 \times 10^{-19} \text{ J}$$

   Conservation of energy:

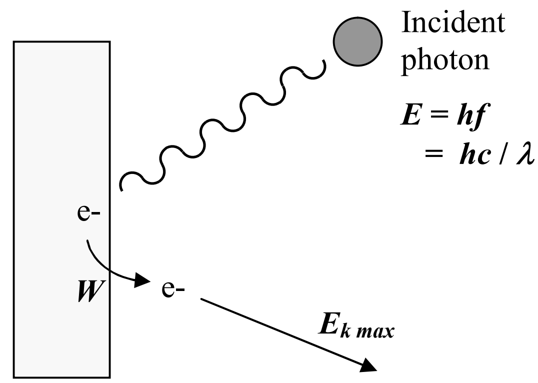
   $E_{Ti} = E_{Tf}$

   $E_{photon} = W + E_{k(\max)}$

   $E_{k(\max)} = E_{photon} - W$

   $= (5.2342 \times 10^{-19} \text{ J}) - (3.6465 \times 10^{-19} \text{ J})$

   $= 1.5877 \times 10^{-19}$ J



   $E_{k(\max)} = q_e V_{stop}$ $V_{stop} = \frac{E_{k(\max)}}{q_e} = \frac{1.5877\times10^{-19} J}{1.60\times10^{-19} C} = 0.99$ V

6. Based on $E_{photon} = \dfrac{hc}{\lambda}$, $E_{photon} \propto \dfrac{1}{\lambda}$ (Inverse relationship).

Thus, the shortest wavelengths will have the greatest photon energies.
We must choose the 400 nm light (i.e. violet) to create the maximum kinetic energy.

$$E_{photon} = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\,Js)(3.00\times10^{8}\,m/s)}{400\times10^{-9}\,m} = 4.9725 \times 10^{-19}\text{ J}$$

$$E_{k(\max)} = \frac{1}{2}mv^2 = 0.5\,(9.11 \times 10^{-31}\text{ kg})\,(7.10 \times 10^{5}\text{ m/s})^2 = 2.2962 \times 10^{-19}\text{ J}$$

Conservation of energy:

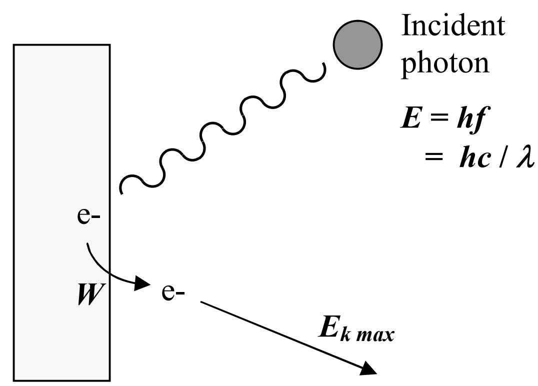
$$E_{Ti} = E_{Tf}$$

$$E_{photon} = W + E_{k(\max)}$$

$$W = E_{photon} - E_{k(\max)}$$

$$= (4.9725 \times 10^{-19}\text{ J}) - (2.2962 \times 10^{-19}\text{ J})$$

$$= 2.6763 \times 10^{-19}\text{ J}$$



Finally, $W = 2.6763 \times 10^{-19}\text{ J} \times \dfrac{1\,eV}{1.60\times10^{-19}\,J} = 1.67\text{ eV}$

7. Determine the number of electrons every second:

$I = \dfrac{q}{t}$ $\qquad q = I\,t = (620 \times 10^{-3}\text{ A})\,(1\text{ s}) = 0.620\text{ C}$

$q = n\,e$ $\qquad n = \dfrac{q}{e} = \dfrac{0.620\,C}{1.60\times10^{-19}\,C} = 3.88 \times 10^{18}$ electrons

Since we assume a one-to-one relationship between low-energy photons and electrons (i.e. one photon can emit only one electron), then the number of photons must be greater than or equal to the number of electrons.

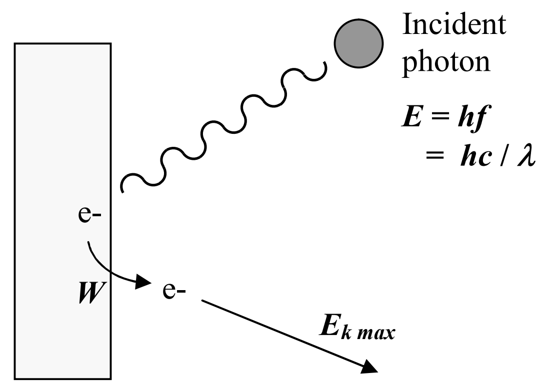
So, the minimum number of photons would be 3.88×10^{18} electrons.

8. a) An increase in intensity (i.e. brightness) means that there are more photons striking the metal surface. More photons means that there will be more electrons emitted. This leads to a greater photocurrent.

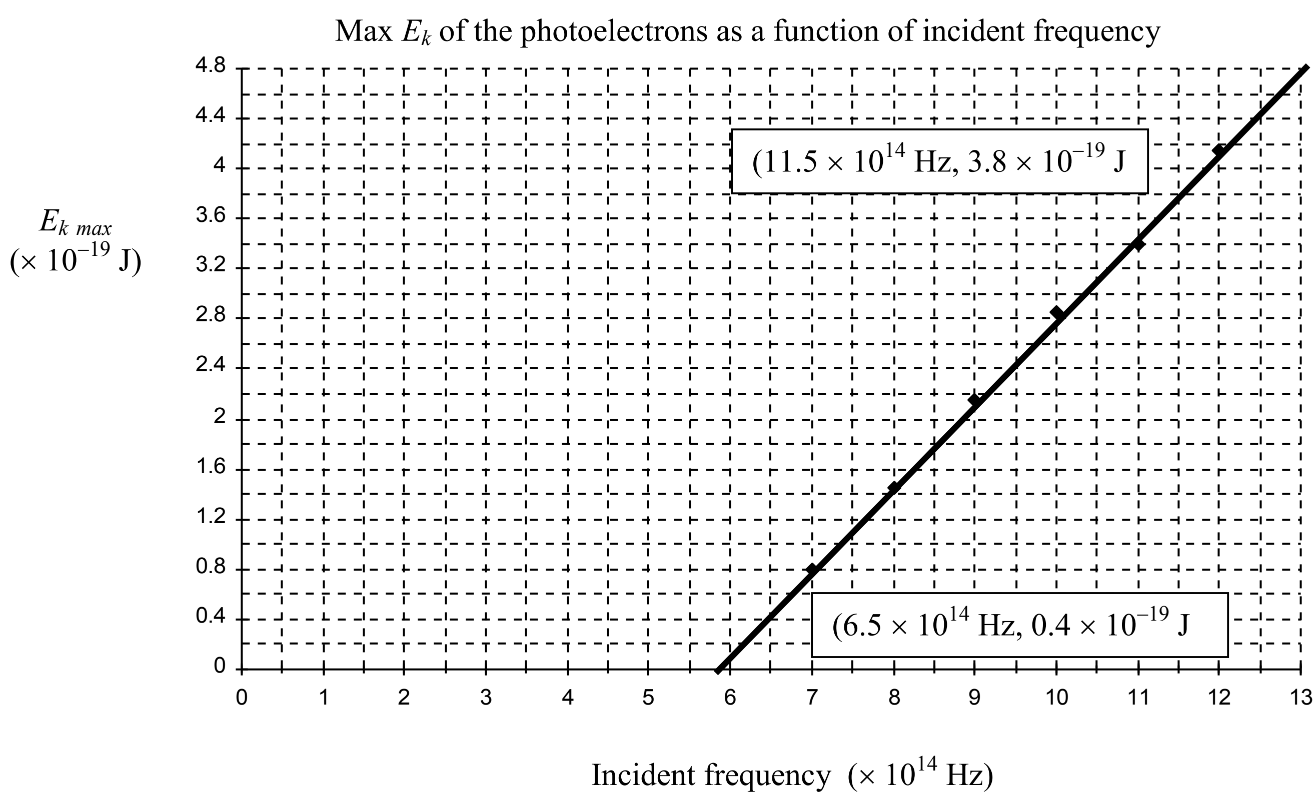
b) Based on $E = hf$, a higher frequency means that the incident photons will have greater energy.

Since the work function is fixed for a metal, if the photons have more energy, then the emitted electrons will have greater kinetic energy.

This means that the electrons will fly off the surface at greater speeds (which means that a greater V_{stop} will be needed to prevent photocurrent).



9.



a) f_o = x-intercept $\approx 5.8 \times 10^{14}$ Hz

b) h = slope = $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{(3.8 - 0.4)\times 10^{-19}\,J}{(11.5 - 6.5)\times 10^{14}\,Hz} = 6.8 \times 10^{-34}\ \dfrac{J}{Hz}$

$= 6.8 \times 10^{-34}\ \dfrac{J}{1/s} = 6.8 \times 10^{-34}$ J·s

c) Be certain to use the experimental values (from the graph).

$W = hf_o$ = (6.8 × 10^{-34} J·s) (5.8 × 10^{14} Hz) = 3.9 × 10^{-19} J

10. $E = pc$ = (4.41 × 10^{-26} kg·m/s) (3.00 × 10^{8} m/s) = 1.323 × 10^{-17} J

$E = hf$ $\quad f = \dfrac{E}{h} = \dfrac{1.323\times 10^{-17}\,J}{6.63\times 10^{-34}\,Js}$ = 2.00 × 10^{16} Hz

11. Find the momentum of the photon:

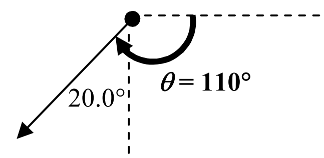
E = 92.0 × 10^{6} eV × $\dfrac{1.60\times 10^{-19}\,J}{1\,eV}$ = 1.472 × 10^{-11} J

$E = pc$ $\quad p = \dfrac{E}{c} = \dfrac{1.472\times 10^{-11}\,J}{3.00\times 10^{8}\,m/s}$ = 4.9067 × 10^{-20} kg·m/s

If the alpha particle has the same momentum,

$p = mv$ $\quad v = \dfrac{p}{m} = \dfrac{4.9067\times 10^{-20}\,kg\cdot m/s}{6.65\times 10^{-27}\,kg}$ = 7.38 × 10^{6} m/s

12. For the Compton Effect, the angle θ is the angle of deflection with respect to its original direction.
So,
$$\theta = 110°$$



$$\Delta\lambda = \frac{h}{mc}(1-\cos\theta) = \frac{6.63\times10^{-34}}{(9.11\times10^{-31})(3\times10^{8})}(1-\cos110°)$$

$= 3.2556 \times 10^{-12}$ m $=$ 3.2556 pm

$\Delta\lambda = \lambda_f - \lambda_i$ $\qquad$ $\lambda_f = \lambda_i + \Delta\lambda$ = 5.80 pm + 3.2556 pm = 9.06 pm

13. If its wavelength increased by 400 am, then $\Delta\lambda = +400 \times 10^{-18}$ m

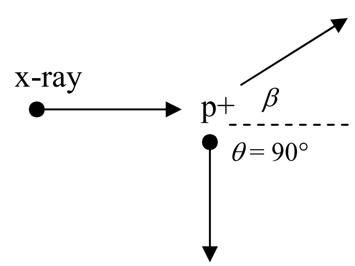
$$\Delta\lambda = \frac{h}{mc}(1-\cos\theta) \qquad \frac{mc\Delta\lambda}{h} = 1-\cos\theta \qquad \cos\theta + \frac{mc\Delta\lambda}{h} = 1$$

$$\cos\theta = 1 - \frac{mc\Delta\lambda}{h} = 1 - \frac{(1.67\times10^{-27})(3\times10^{8})(400\times10^{-18})}{6.63\times10^{-34}} = 0.6977$$

$$\theta = \cos^{-1}(0.6977) = 45.8°$$

14. a) $\Delta\lambda = \frac{h}{mc}(1-\cos\theta) = \frac{6.63\times10^{-34}}{(1.67\times10^{-27})(3\times10^{8})}(1-\cos90°)$

$= 1.323 \times 10^{-15}$ m = 1.323 fm



$\Delta\lambda = \lambda_f - \lambda_i$

$\lambda_f = \lambda_i + \Delta\lambda$ = 4 fm + 1.323 fm = 5.323 fm

$v = f\lambda$ $\qquad$ $f = \frac{v}{\lambda} = \frac{c}{\lambda} = \frac{3\times10^{8}}{5.323\times10^{-15}} = 5.64 \times 10^{22}$ Hz

b) **Conservation of Energy**:

$$E_{Ti} = E_{Tf} \qquad E_{xray} = E'_{xray} + E_{k(p+)}$$

$$E_{k(p+)} = E_{xray} - E'_{xray} \qquad E_{k(p+)} = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_f}$$

$$E_{k(p+)} = \frac{(6.63\times10^{-34})(3\times10^{8})}{4\times10^{-15}} - \frac{(6.63\times10^{-34})(3\times10^{8})}{5.323\times10^{-15}} = 1.24 \times 10^{-11}\ \text{J}$$

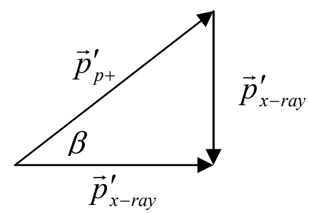
c) $$p_{xray} = \frac{E_{xray}}{c} = \frac{h}{\lambda_i} = \frac{6.63\times10^{-34}}{4\times10^{-15}} = 1.6575 \times 10^{-19}\ \text{kg·m/s}$$

$$p'_{xray} = \frac{E'_{xray}}{c} = \frac{h}{\lambda_f} = \frac{6.63\times10^{-34}}{5.323\times10^{-15}} = 1.2455 \times 10^{-19}\ \text{kg·m/s}$$

Conservation of momentum:

$$\vec{p}_{xray} = \vec{p}'_{xray} + \vec{p}_{p+}$$

This leads to the vector triangle shown.



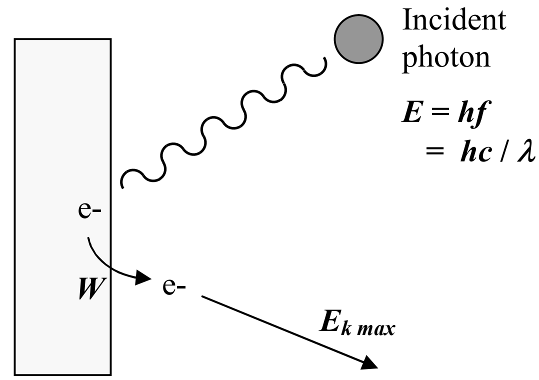
$$c^2 = a^2 + b^2 \qquad p_{p+} = \sqrt{(1.6575\times10^{-19})^2 + (1.2455\times10^{-19})^2}$$

$$= 2.07 \times 10^{-19}\ \text{kg·m/s}$$

$$\tan\beta = \frac{p'_{xray}}{p_{xray}} = \frac{1.2455\times10^{-19}}{1.6575\times10^{-19}} \qquad \beta = \tan^{-1}(0.7515) = 36.9°$$

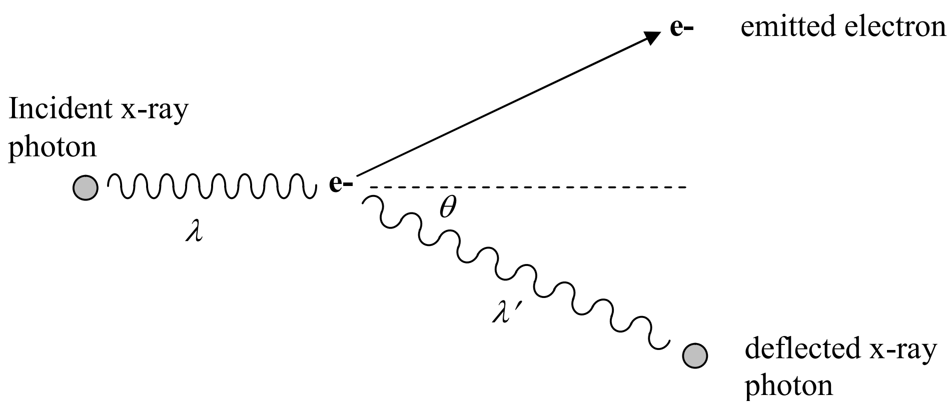
15. Photoelectric Effect

The incident photon is fully absorbed by the electron, the electron is emitted, and the electron keeps the rest of the energy as kinetic energy



Compton Effect

The incident x-ray photon is only partially absorbed, emitting the electron and giving the electron kinetic energy. The rest of the photon energy is reemitted as an x-ray photon with less energy (lower f, longer λ).



Same: Both absorb energy from a photon and an electron is emitted.

Different: Photoelectric effect involves complete absorption of the photon, while in the Compton Effect, there is only partial absorption. That is, in the Compton Effect, the photon behaves more like a particle.